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Exploring Common Errors, Challenges, and Strategies of Solving Limits in Calculus

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ABSTRACT

This paper presents findings from a case study examining undergraduate students' common errors when solving limit problems, the challenges encountered, and the strategies employed in enhancing their understanding and performance in this topic. A mixed-methods approach was utilised, combining quantitative and qualitative data collection. Common errors were assessed by analysing students' assessment work solutions, while challenges in learning limits and strategies for improvement were investigated through surveys conducted among students and lecturers. Additional qualitative feedback from lecturers provided insights into the challenges and effective strategies for improvement. The results identified three common types of errors, where conceptual errors (misunderstandings of the fundamental principles and concepts underlying limit problems) contributed to the highest percentage of errors at 67.6%, followed by procedural errors (incorrect steps or algorithms in the mathematical process of solving limit problems), and factual errors (mistakes in recalling or applying mathematical facts related to limits), with 18.9% and 13.5% error percentage, respectively. While most students expressed positive feelings towards learning limits of functions, a significant number reported being hesitant or moderately confident when solving limit problems. The study identified insufficient pre-requisite knowledge, conceptual confusion, as well as procedural and factual inaccuracies as the most significant challenges students face in understanding limits, with consulting resources, revisiting foundational concepts, and collaborative learning being the most effective strategies for improvement. Frequent practice, constructive feedback, and addressing misconceptions and common errors were emphasised as critical methods to enhance students' comprehension and performance in solving limit problems. Limitations of the study included a small sample size and a focus on specific programmes, prompting recommendations for broader studies with diverse samples and exploration of teaching interventions in future research.

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1.0 INTRODUCTION

Mastering the concept of limits in calculus is crucial due to its significant relevance in everyday applications. Understanding limits enables us to analyse and solve complex problems involving trends, approximations, and the behaviour of systems over time. Despite its importance, many undergraduate students struggle to grasp this concept effectively. Studies have shown that students encounter various challenges related to limits, including misunderstandings of fundamental principles and concepts, errors in executing the correct procedures, and mistakes in applying mathematical facts. These challenges often stem from inadequate conceptual understanding, insufficient pre-requisite knowledge, memorising procedures without understanding the concepts behind them, and many other factors.

Although past studies have delved into the root causes of these challenges and proposed strategies to address them, gaps remain in understanding the specific challenges and errors faced by undergraduate students in this area. This study aimed to bridge this gap and add significant value to the existing knowledge. A detailed case study was conducted to identify the common errors committed by students, challenges, and strategies employed by both students and lecturers to improve understanding and performance in solving the limit problems. In specific, this paper is guided by the following research questions:

1. What are the common errors committed by students when solving limits of functions?
2. What challenges do students face in understanding the concept of limits in calculus?
3. What strategies are employed by students and lecturers to enhance understanding and improve performance in solving limit problems?

The findings could help educators design and implement more effective teaching strategies to address specific errors and challenges, thereby enhancing students' understanding in calculus, particularly in solving limits of functions. By employing targeted strategies, educators can strengthen students' conceptual understanding of limits, which is essential for success in advanced mathematical topics and their practical applications. Without solid foundational concepts, students may struggle with more complex ideas, leading to persistent difficulties in their mathematical education.

2.0 LITERATURE REVIEW

2.1 Types of Errors in Solving Limits

When solving limits of functions, students often make three primary types of errors: conceptual errors, procedural errors, and factual errors (Brown et al., 2016; Lai, 2012; Giri & Gowramma, 2021). Conceptual errors arise from misunderstandings of fundamental concepts and principles related to limits (Brown et al., 2016; Giri & Gowramma, 2021; Salido et al., 2014). These errors include misidentifying limits, applying incorrect limit theorems, or failing to simplify expressions before substitution (Brown et al., 2016; Lai, 2012; Villavicencio, 2023).

Procedural errors arise from inadequate problem-solving strategies, such as skipping steps, misinterpreting required procedures, or mishandling algebraic manipulations (Lai, 2012; Brown et al., 2016; Baidowi et al., 2023). These errors often due to students memorizing steps without without fully understanding the concepts, making them prone to mistakes when faced with even slight modifications in problem structure (Lai, 2012; Denbel, 2014; Nurhayati and Retnowati, 2019). Struggles with trigonometric identities, factoring expressions, and rationalising fractions further complicate problem-solving efforts (Baidowi et al., 2023).

Factual errors involve mistakes in applying mathematical facts related to limits, including incorrect use of terminology, basic arithmetic errors, or improper use of formulas (Brown et al., 2016; Giri & Gowramma, 2021; Lai, 2012). Misuse of mathematical symbols, omission of essential notations like parentheses, or failure to include limit notation during calculations frequently leads to incorrect solutions (Baidowi et al., 2023; Brown et al., 2016; Lai, 2012).While these errors are often linked to carelessness (Lai, 2012), they may also reflect deeper gaps in foundational mathematical knowledge that significantly impact students' ability to solve limit problems accurately (Brown, et al., 2016; Lai, 2012). Understanding the root causes of these errors allows educators to tailor interventions that address students' specific learning difficulties.

2.2 Challenges in Solving Limits

Students encounter various challenges that contribute to errors in solving limits of functions. From literature, these challenges can be categorised into five main areas: insufficient pre-requisite knowledge, conceptual confusion, procedural inaccuracy, factual inaccuracies, and inadequate attitude and motivation.

A major hurdle is the lack of pre-requisite knowledge in algebra and trigonometry, which hinders students' ability to correctly apply limit theorems (Salido et al., 2014; Lai, 2012). Salido et al. (2014) found that students with weak algebraic foundations struggle with essential operations such as factoring, rationalization, and function manipulation, leading to difficulties in evaluating limits. Lai (2012) further emphasised that students who lack proficiency in algebra and trigonometry often fail to recognize patterns in limit problems, making it difficult for them to apply problem-solving strategies effectively.

Another major challenge is conceptual confusion which includes misinterpreting the nature of limits (Denbel, 2014; Salido et al., 2014). Denbel (2014) found that many students mistakenly equate a function's limit with its function value. Similarly, Salido et al. (2014) observed that students often rely on direct substitution instead of applying appropriate limit theorems, resulting in incorrect conclusions.

Procedural inaccuracy occurs when students skip steps, misapply algorithms, or rush through problems due to incomplete understanding or prioritizing speed over accuracy (Brown et al., 2016; Lai, 2012; Salido et al., 2014). Lack of focus during lessons further contributes to careless mistakes and further impacting students' ability to solve limit problems systematically.

Factual inaccuracy, though often considered minor, significantly impact student performance (Denbel, 2014; Baidowi et al., 2023). Denbel (2014) found that weak retention of mathematical properties leads students to assume incorrect limit values. Likewise, Baidowi et al. (2023) reported that students frequently misuse notation, omit necessary limit expressions, or make arithmetic errors, leading to incorrect results.

Beyond cognitive challenges, attitude and motivation play a crucial role in solving limits. Misconceptions, fragmented understanding of limit definitions, and low confidence discourage students from engaging in problem-solving and persisting in mastering limits (Giri & Gowramma, 2021; Villavicencio, 2023).

2.3 Strategies to Improve Understanding in Solving Limits

Addressing errors and challenges in solving limits requires tailored instructional strategies. Educators can focus on correcting conceptual misunderstandings by reinforcing fundamental principles and providing clear explanations of definitions and theorems (Villavicencio, 2023; Nurhayati & Retnowati, 2019). Procedural issues can be minimized through structured problem-solving exercises that emphasize step-by-step accuracy (Lai, 2012; Brown et al., 2016). Factual errors can be reduced by improving students' attention to detail, encouraging careful verification of steps, and fostering a deeper understanding of mathematical symbols and terminology (Giri & Gowramma, 2021; Brown et al., 2016). Understanding the errors and challenges enables the development of innovative lesson plans that promote deeper comprehension and retention of material (Nurhayati & Retnowati, 2019).

Regular practice, detailed feedback, and discussion of common mistakes help students develop better problem-solving techniques and reinforce proper methodologies (Baidowi et al., 2023). Collaborative learning, such as peer discussions and group problem-solving sessions, enhances understanding by promoting idea exchange and mutual learning (Villavicencio, 2023). Moreover, integrating technology tools like graphing calculators and interactive software provides visual aids and practical resources to strengthen comprehension and application of limits (Nurhayati & Retnowati, 2019; Denbel, 2014).

Effective strategies not only improve students' performance in calculus but also boost their confidence and motivation in mathematics (Lai, 2012; Villavicencio, 2023; Brown et al., 2016).

While many studies have focused on students' challenges, fewer have simultaneously assessed teaching and learning strategies and their effectiveness in addressing these challenges. By bridging this gap, this study provides a dual perspective that enhances both students' learning outcomes and teaching efficacy, fostering a more comprehensive approach to mathematics education.

2.4 Importance of Addressing Errors, Challenges, and Strategies in Solving Limits

A comprehensive approach to address errors, challenges, and strategies in solving limits is crucial for fostering students' mathematical reasoning and problem-solving skills. While conceptual and procedural have been the focus of research, factual errors also significantly hinder students' performance (Lai, 2012, Giri & Gowramma, 2021; Brown et al., 2016). Thus, focusing solely on conceptual and procedural errors may lead to inefficient teaching practices that overlook students' foundational misunderstandings (Lai, 2012; Villavicencio, 2023).

A holistic instructional approach that targets all error types is essential for building a strong mathematical foundation and ensuring holistic improvement in calculus learning (Villavicencio, 2023). Tailored instructional strategies that support students with diverse backgrounds and prior knowledge, are vital for promoting equity and enabling all learners to succeed This inclusive approach not only improves students' performance in mathematics but also boosts their confidence, motivation, and readiness to excel in related fields (Nurhayati & Retnowati, 2019; Jameson et al., 2023; Brown et al. 2016).

Unlike prior studies that examined errors, challenges, and strategies separately, this study integrates these aspects into a single comprehensive framework, offering a holistic approach to enhancing teaching and learning practices of solving limits of functions in calculus.

3.0 METHODOLOGY

3.1 Research Design

As described in Figure 1, this study involved two phases and employed a mixed-methods research design to provide a comprehensive understanding of the errors, challenges, and strategies related to solving limits of functions in calculus. The mixed-methods approach combines quantitative and qualitative analyses, enabling a deeper and broader understanding while enhancing the confidence, validity, and reliability of findings (Adu et al., 2022).

In the first phase, the assessment work solutions from 51 undergraduate students were analysed using error analysis to identify error patterns in solving limits of functions. Error analysis is a systematic process of reviewing a student's work and determining whether an error pattern exists (Brown et al., 2016; Lai, 2012). Error analysis is an effective method for identifying patterns of mathematical errors (Brown et al., 2016). The quantitative analysis involved evaluating students' marks to determine the prevalence and distribution of errors, thereby identifying areas where students struggle the most. Concurrently, qualitative analysis was performed by scrutinising the patterns and causes of errors in students' problem-solving processes.

In the second phase, self-designed surveys were distributed to both students and lecturers to gather data on the challenges faced by students in solving limit problems and the strategies employed to enhance their understanding and performance. The survey's instrument was developed based on an extensive literature review of misconceptions in calculus and error analysis studies by Brown et al (2016). The inclusion of both closed and open-ended questions ensured a balanced approach, with open-ended items providing deeper insights into the root causes of errors and effective strategies for addressing them.

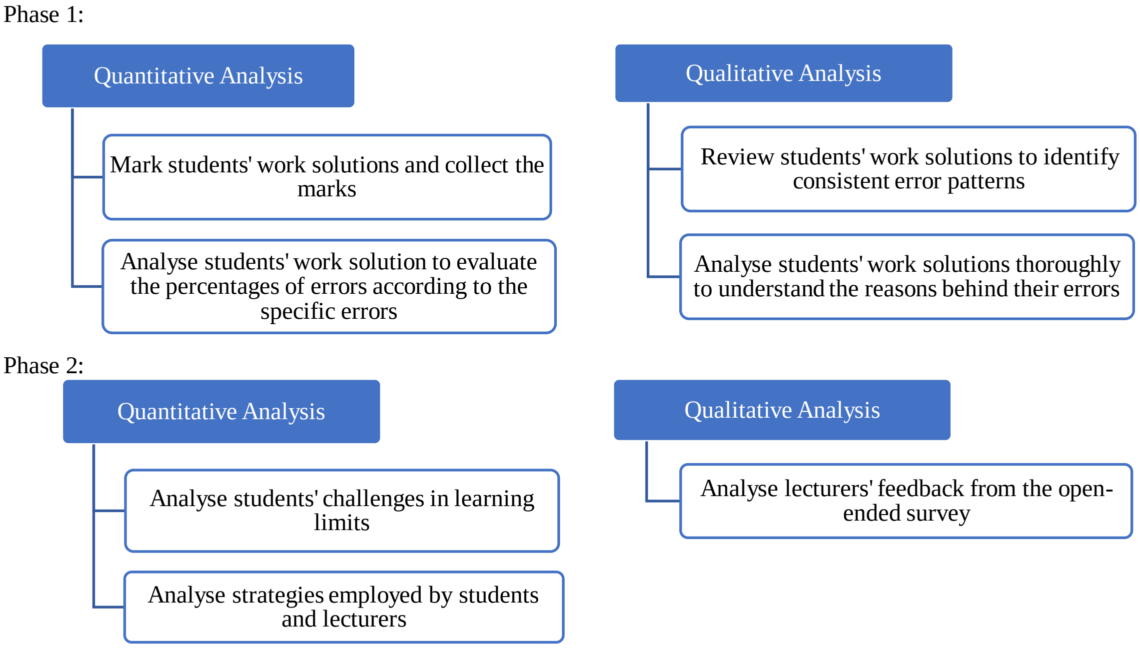


Fig. 1. A research design that combined both quantitative and qualitative analysis

3.2 Research Instrument

As outlined in Table 1, this study focused on three questions related to solving limits of rational functions, with a total of 11 marks allocated. These questions required different solution methods, such as factorisation, rationalisation, and squeezing theorem. These methods were chosen based on their significance in limit solving and their documented role in students' learning difficulties. Students' marks were analysed quantitatively to evaluate the percentage of errors made and categorised as conceptual, procedural, and factual errors based on the framework by Brown et al. (2016).

Surveys were used to supplement the error analysis by capturing students' learning experiences and lecturers' pedagogical approaches. The student survey comprised two sections. Section A gathered demographic profiles of the students, while Section B focused on students' comfort and confidence levels in learning limits and their preferred learning strategies. Meanwhile, the lecturer survey comprised three sections. Section A gathered demographic profiles of the lecturers, and Section B collected insights into the students' challenges, their impact on performance, and the teaching strategies employed to enhance students' understanding in solving limits of functions. Open-ended questions in Section C enabled lecturers to share detailed observations of common student errors, challenges, and potential improvements to teaching strategies. By combining structured and open responses, the survey provided comprehensive data, enabling a deeper exploration of underlying issues.

To ensure reliability, the internal consistency of survey items in Section B for both surveys was examined using Cronbach's Alpha, which ranges from 0 to 1. As shown in Table 2, the resulting reliability coefficient ranged from 0.733 to 0.948 (>0.70), confirming the instrument's reliability in measuring the intended constructs.

Table 1: Research instrument with three questions of solving limit problems

Question	Question Description	Allocation of Marks	Solution
Q1	Solve $\lim_{x\to 3}\frac{4x-12}{x^2+x-12}$	3 marks	Factorise both numerator and denominator: $\lim_{x\to 3}\frac{4(x-3)}{(x-3)(x+4)}$ Then, simplify and apply direct substitution: $=\lim_{x\to 3}\frac{4}{x+4}=\frac{4}{3+4}=\frac{4}{7}$
Q2	Solve $\lim_{x\to -4}\frac{\sqrt{x+8}-2}{x+4}$	4 marks	Rationalise the numerator: $\lim_{x\to -4}\frac{\sqrt{x+8}-2}{x+4}.\frac{\sqrt{x+8}+2}{\sqrt{x+8}+2}$ Then, simplify: $=\lim_{x\to -4}\frac{(\sqrt{x+8})^2-(2)^2}{(x+4)(\sqrt{x+8}+2)}$ $=\lim_{x\to -4}\frac{(x+8)-4}{(x+4)(\sqrt{x+8}+2)}$ $=\lim_{x\to -4}\frac{x+4}{(x+4)(\sqrt{x+8}+2)}$ $=\lim_{x\to -4}\frac{1}{\sqrt{x+8}+2}$ Hence, apply direct substitution: $=\frac{1}{\sqrt{-4+8}+2}=\frac{1}{4}$
Q3	Solve $\lim_{x\to 0}\frac{2x+sin3x}{x}$	4 marks	Multiply the function with $\left(\frac{3}{3}\right)$: $\lim_{x\to 0}\frac{2x+sin3x}{x}.\left(\frac{3}{3}\right)=3\lim_{x\to 0}\frac{2x+sin3x}{3x}$ Separate the terms and simplify: $=3\left[\lim_{x\to 0}\frac{2}{3}+\lim_{x\to 0}\frac{sin3x}{3x}\right]$ Hence, apply the squeezing theorem: $=3\left[\frac{2}{3}+1\right]=3\left[\frac{5}{3}\right]=5$

Table 2. Cronbach's alpha

Constructs	No. of Items	Cronbach's Alpha
Strategies to improve understanding of limits (Student Survey)	5	0.733
Challenges faced by students in learning limits (Lecturer Survey)	5	0.891
Impact of challenges on students' performance (Lecturer Survey)	5	0.846
Strategies to improve students' understanding of limits (Lecturer Survey)	10	0.948

3.3 Research Sample

The sample of the study comprised 51 undergraduate students enrolled in Calculus 1 and all 9 lecturers teaching the course during the October 2023 – February 2024 semester at Universiti Teknologi MARA (UiTM) Sarawak. The students, representing a mix of science-related academic programmes, accounted for 27.3% of the total population of 187 students. This sample size aligns with the recommendation of Krejcie and Morgan (1970) for adequate statistical power and generalisability. The diversity within the sample, encompassing feedback from both students and lecturers, allowed for a multi-perspective exploration of the difficulties in learning limits.

4.0 RESULTS

4.1 Students' Demographic Profiles

Table 3 shows the demographic profiles of the student respondents. 29 (56.9%) of the students were males, and the remaining 22 (43.1%) were females. With regards to the study programme, 20 (39.2%) of the students were from Diploma in Civil Engineering, 18 (35.3%) were from Diploma in Applied Science, and 13 (25.5%) were from Diploma in Electrical Engineering. All of them achieved at least a B+ in SPM Mathematics, with the majority (98.0%) scoring an A. In contrast, only 5.9% of them scored an A, while the majority (74.5%) scored C+ or below in SPM Additional Mathematics. The results highlight that while students excel in Mathematics, Additional Mathematics poses a greater challenge.

Table 3. Profiles of the student respondents

Profiles	Total
Gender (n=51)	
Male	29 (56.9%)
Female	22 (43.1%)
Programme of Study (n=51)	
Diploma in Civil Engineering	20 (39.2%)
Diploma in Applied Science	18 (35.3%)
Diploma in Electrical Engineering	13 (25.5%)
SPM Mathematics Result (n=51)	
A+	7 (13.7%)
A	41 (80.4%)
A-	2 (3.9%)
B+	1 (2.0%)
SPM Additional Mathematics Result (n=51)	
A	2 (3.9%)
A-	1 (2.0%)
B+	2 (3.9%)
B	8 (15.7%)
C+	9 (17.6%)
C	12 (23.5%)
D	17 (33.3%)

4.2 Analysis of Errors in Solving Limits

Type and Percentage of Errors

In this study, the marks obtained by the 51 undergraduate students while solving questions on limits of functions in a calculus course were used as a measurement to evaluate the percentages of errors. Table 4 and Table 5 show the specific errors committed by the students and their percentages, respectively.

Table 4. Types of errors

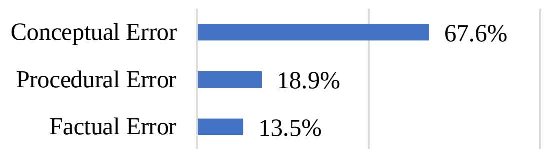
Student No	Type of Errors Q1	Q2	Q3	Student No	Type of Errors Q1	Q2	Q3
1	C	C	X	27		X	
2				28			
3				29	F		P
4				30			
5				31		X	
6			C	32			
7			C	33	F	C	
8				34		P	
9		C		35			
10			C	36	F	P	
11				37			C
12				38			C
13	C		C	39			
14				40		C	
15				41		P	C
16		P		42		C	C
17				43			
18				44			C
19			P	45	C		
20				46		C	C
21				47			C
22				48			C
23		C	C	49			
24		F		50			C
25				51	F	P	
26		C					

Note: C=Conceptual Error; P=Procedural Error; F=Factual Error; X=No Attempt; ▒ = Correct Answer

The results show that three types of common errors were observed, where conceptual errors contributed to the highest error percentage of 67.6%. This was followed by procedural errors and factual errors, with 18.9% and 13.5% of error percentages, respectively.

Table 5. Percentage of errors

Type of Errors	Frequency	Percentage
Conceptual	25	67.6%
Procedural	7	18.9%
Factual	5	13.5%
Total Error	37	

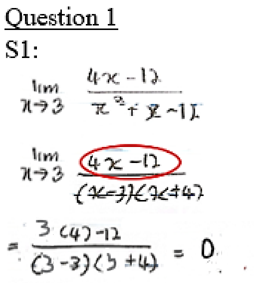
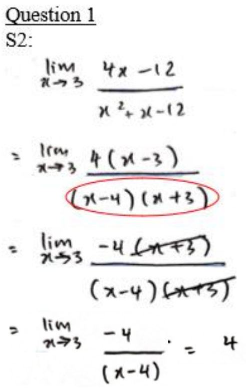
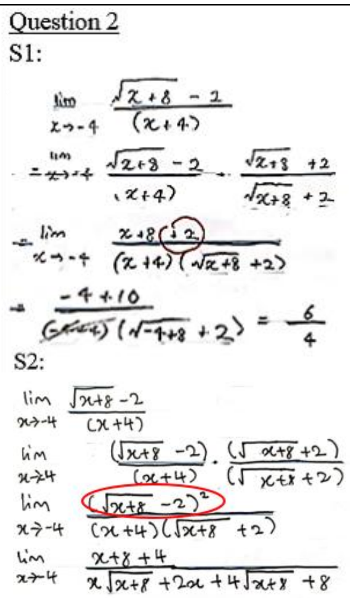


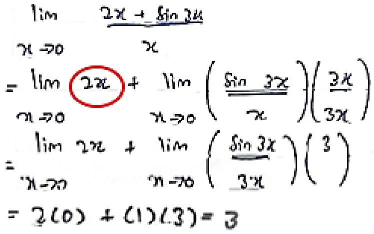
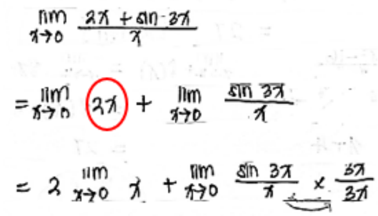
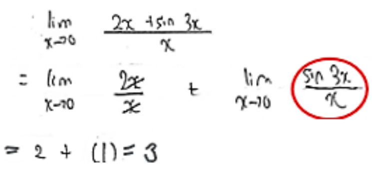
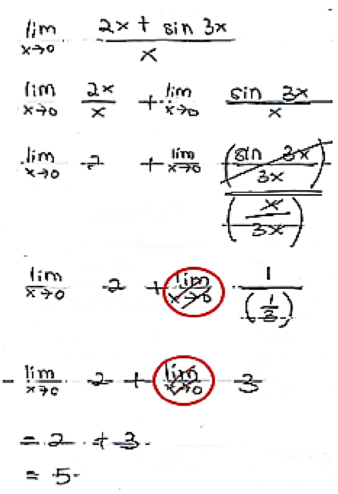
Samples of Student Errors

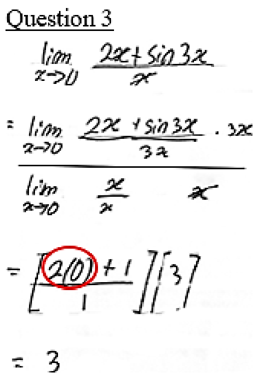
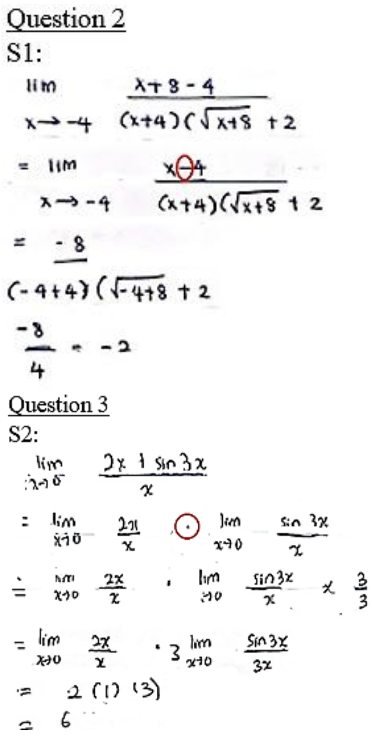
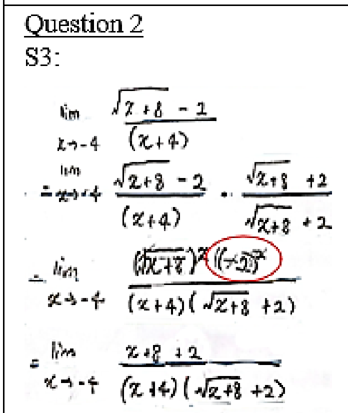
A review of students' work solutions resulted in some consistent error patterns or common errors committed by students when solving limits of functions, as outlined in Table 6. Conceptual errors include mistakes like failing to factorise functions completely or correctly before applying direct substitution, incorrectly expanding and separating the terms, misapplying theorems, and incorrectly using limit notation. These errors may occur due to students' misunderstanding of fundamental concepts and principles in solving limits.

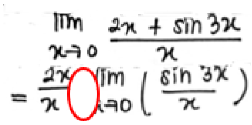
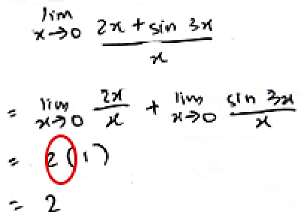
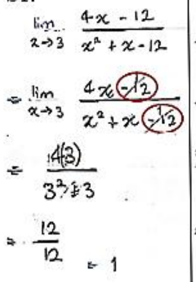
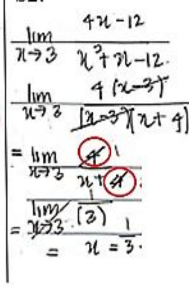
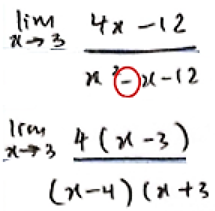
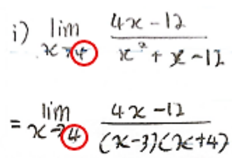
Procedural errors include mistakes like failing to follow the correct steps or procedures in solving limits and dropping or misplacing signs. These mistakes arise from carelessness or less attention to detail during the execution of steps. On the other hand, factual errors involve mistakes in basic arithmetic and number facts, such as incorrect cancelations or misapplication of expressions, due to a lack of knowledge in fundamental arithmetic facts or properties. Copying incorrect functions also constitutes a factual error. These errors highlight the challenges students face in understanding and applying the correct concepts and procedures, while minimizing careless when solving limits of functions.

Table 6. Samples of errors

Type of Errors		Sample of Errors	Observation
Conceptual Errors	Incorrect factorisation S1: ($4x – 12$) was not factorised S2: ($x^2 + x – 12$) was factorised wrongly		The students did not factorise the function completely before applying direct substitution, possibly due to a lack of understanding of the principles for solving limits, particularly the need to simplify expressions to avoid indeterminate forms.
			The students inserting the wrong signs during factorisation reflects a misunderstanding of the principles behind factorisation (e.g., not knowing how the signs should be assigned during factorisation).
	Incorrect expansion of the terms S1& S2: $(\sqrt{x+8}-2)(\sqrt{x+8}+2)$ was wrongly expanded		The students incorrectly expanded the terms due to misapplying the rules of algebra, reflecting errors in the execution.

Conceptual Errors	Incorrect separation of the terms S1&S2: *2x* was separated without the denominator *x*	Question 3 S1:  S2: 	The students incorrectly separated the terms in a fraction by writing the first term without the denominator, indicating a misunderstanding of the fundamental property of fractions, where each term in the numerator must be divided by the denominator when separating them.
	Incorrect use of the theorem: The squeezing theorem was directly applied without changing the denominator *x* to *3x*	Question 3 	The student incorrectly applied the Squeezing Theorem due to a misunderstanding of its specific conditions or the logic behind the theorem.
	Inappropriate writing of *lim* when executing limit	Question 3 	The student continued to write "*lim*" after substituting the value, indicating a lack of understanding that once the limit process was completed and the value had been substituted, the limit notation should no longer be used.

Procedural Errors	Inability to follow the correct steps or procedures		The student neglected to follow the appropriate steps before directly substituting the value.
	Misplacement of signs S1: Inserted the sign ‘-’ instead of ‘+’ S2: Inserted the sign ‘.’ instead of ‘+’ S3: Wrongly placed the negative sign inside the bracket and mistakenly expanded the expression		The students accidentally wrote the wrong sign due to carelessness or a simple mistake.
			The student incorrectly placed the negative sign inside the bracket and mistakenly expanded the expression.

Procedural Errors	Dropping of sign S1&S2: forgot to insert the addition sign	Question 3 S1:  S2: 	The students neglected to write the necessary sign after separating the terms due to a lack of thoroughness in their mathematical operations.
Factual Errors	Inability to master the basic number facts S1: The value of *12* was wrongly cancelled S2: The value of *4* was wrongly cancelled	Question 1 S1:  S2: 	The students did the wrong cancellation to the value or expression due to a lack of knowledge of fundamental arithmetic facts or properties.
	Incorrect copying of the function S1: the sign '+' became '–' S2: approaching value '3' became '4'	Question 1 S1:  S2: 	The students incorrectly copied the sign due to carelessness or lack of attention.

4.3 Students' Comfort and Confidence Levels

The results of the students' survey on their comfort and confidence levels in learning the limits of functions were reported in Table 7. While 39.2% of the students felt "comfortable" and 33.3% felt "moderately comfortable" in learning limits of functions, a smaller but significant portion felt "uncomfortable" (21.6%) or "very uncomfortable" (2.0%) (Mean=3.22, S.D.=.901). Nevertheless, while

49.0% of the students expressed "moderately confident" and 31.4% expressed "more confident" in solving limit problems, a smaller group remained uncertain, with 15.7% being "less confident" and another 3.9% being "not confident at all" (Mean=3.08, S.D.=.796).

Table 7. Students' comfort and confidence levels in learning limits of functions

	Very un-comfort	**Un-comfort**	**Moderate**	**Comfort**	**Very comfort**	**Mean**	**S.D.**
Comfort level in learning limits of functions	1 (2.0%)	11 (21.6%)	17 (33.3%)	20 (39.2%)	2 (3.9%)	3.22	.901
	Not confident at all	**Less confident**	**Moderate**	**More confident**	**Very confident**	**Mean**	**S.D.**
Confidence level in solving limit problems	2 (3.9%)	8 (15.7%)	25 (49.0%)	16 (31.4%)	0 (0.0%)	3.08	.796

These findings suggest that while most students felt positively inclined to learn limit of functions, there was a significant portion that remained either hesitant or moderately confident in solving limit problems. These insights highlight the importance of tailored instructional strategies to boost the confidence in students who are struggling with the concept and thus enhance their overall comfort and understanding in learning limits of functions.

4.4 Students' Strategies to Improve Understanding of Limits

Table 8 presents the students' feedback on the strategies to improve their understanding of limits. The most frequently recommended approach was consulting resources, whether through notes, online tutorials, or guidance from lecturers (Mean=3.73, S.D.=.874). Working with peers (Mean=3.51, S.D.=.809) and revisiting foundational concepts (Mean=3.51, S.D.=.834) are also commonly employed, providing a balance of individual and collaborative learning approaches. Practising additional problems (Mean=3.43, S.D.=.878) is moderately utilised, while asking specific questions in class or office hours (Mean=2.57, S.D.=.878) is the least favoured strategy. These findings indicate that while students are proactive in seeking external help and reinforcing foundational knowledge, they are less likely to directly engage with instructors to clarify challenges and seek direct help.

Table 8. Students' strategies to improve their understanding in learning the limits of functions

Strategy	**Never**	**Rarely**	**Some-times**	**Often**	**Always**	**Mean**	**S.D.**
Consulting resources	0 (0.0%)	3 (5.9%)	19 (37.3%)	18 (35.3%)	11 (21.6%)	3.73	.874
Working with peers	0 (0.0%)	5 (9.8%)	20 (39.2%)	21 (41.2%)	5 (9.8%)	3.51	.809
Revisiting foundational concepts	0 (0.0%)	5 (9.8%)	21 (41.2%)	19 (37.3%)	6 (11.8%)	3.51	.834
Practising additional problems	0 (0.0%)	8 (15.7%)	18 (35.3%)	20 (39.2%)	5 (9.8%)	3.43	.878
Asking specific questions in class	4 (7.8%)	22 (43.1%)	18 (35.3%)	6 (11.8%)	1 (2.0%)	2.57	.878

Further analysis of the students' open feedback revealed that consulting resources is the most preferred strategy for learning limits as it provides flexibility in addressing specific challenges and clarifies concepts. Repeated practice involving solving multiple exercises strengthens memory, improves method selection, and fosters structured approaches to solving problems. Reviewing and creating personalised notes simplifies understanding, while practising past-year questions enhances familiarity with exam formats. Collaborative strategies, such as group discussions and peer study, facilitate idea exchange, allow students to learn from others, and verify answers, thereby enhancing problem-solving skills and addressing knowledge gaps. External resources, including YouTube tutorials, structured exercise materials, and technology tools like graphing calculators, further support independent learning by enabling visualisation and solution validation. Collectively, these strategies highlight the importance of combining individual effort, collaborative learning, and diverse resources to overcome challenges in understanding limits.

4.5 Lecturers' Demographic Profiles

Table 9 shows the demographic profiles of the lecturer respondents. The survey included 9 lecturers, of which 7 were female (77.78%), and 2 were male (22.22%). The majority of respondents (4 lecturers, 44.44%) were in the 40 to 49 years old age group, followed by 3 lecturers (33.33%) aged 30 to 39 years, and 1 lecturer (11.11%) in the 50 years old and above age group. The majority (5 lecturers, 55.56%) had more than 6 semesters of experience in teaching Calculus 1, while 3 lecturers (33.33%) reported 3 to 6 semesters, and only 1 lecturer (11.11%) had less than 3 semesters of experience. This profile suggests that the respondents were predominantly experienced female educators.

Table 9. Profiles of the lecturer respondents

Profiles	Total
Gender (n=9)	
Male	2 (22.22%)
Female	7 (77.78%)
Age Range (n=9)	
30 to 39 years old	3 (33.33%)
40 to 49 years old	4 (44.44%)
50 years old and above	1 (11.11%)
Teaching Experiences in Calculus 1 (n=9)	
More than 6 semesters	5 (55.56%)
3 to 6 semesters	3 (33.33%)
Less than 3 semesters	1 (11.11%)

4.6 Students' Challenges in Learning Limits

Table 10 shows the lecturers' feedback regarding challenges faced by students in learning limits of functions. The greatest challenge was insufficient pre-requisite knowledge, with 77.7% of lecturers identifying it as occurring "very often" and "often" (Mean=4.22, S.D.=.833). Conceptual confusion was also frequently noted (Mean=3.89, S.D.=.782), followed closely by procedural inaccuracy (Mean=3.78, S.D.=.833), factual inaccuracy (Mean=3.78, S.D.=.972), and inadequate attitude and motivation (Mean=3.56, S.D.=1.130).

Table 10. Challenges faced by students in learning limits of functions

Challenges	Very Often	Often	Sometimes	Occasionally	Not Often	Mean	S.D.
Insufficient Pre-requisite Knowledge	4 (44.4%)	3 (33.3%)	2 (22.2%)	0 (0%)	0 (0%)	4.22	.833
Conceptual Confusion	2 (22.2%)	4 (44.4%)	3 (33.3%)	0 (0%)	0 (0%)	3.89	.782
Procedural Inaccuracy	2 (22.2%)	3 (33.3%)	4 (44.4%)	0 (0%)	0 (0%)	3.78	.833
Factual Inaccuracy	2 (22.2%)	4 (44.4%)	2 (22.2%)	1 (11.1%)	0 (0%)	3.78	.972
Inadequate Attitude & Motivation	2 (22.2%)	3 (33.3%)	2 (22.2%)	2 (22.2%)	0 (0%)	3.56	1.130

4.7 Impact of Challenges on Students' Performance

Regarding the impact on students' performance as shown in Table 11, insufficient pre-requisite knowledge (Mean=4.33, S.D.=1.00) was identified as the most impactful challenge, which was rated as "very high" and "high" impact by 88.8% of the lecturers. Procedural inaccuracy (Mean=4.22, S.D.=.833) and conceptual confusion (Mean=4.11, S.D.=.782) followed closely, each rated as having a "very high" and "high" impact by 77.7% of the lecturers. Meanwhile, factual inaccuracy (Mean=3.78, S.D.=.972) and inadequate attitude and motivation (Mean=3.56, S.D.=1.014) were generally seen as less impactful, though still a concern.

Table 11. Impact of challenges on students' performance

Challenges	Very High	High	Medium	Low	Very Low	Mean	S.D.
Insufficient Pre-requisite Knowledge	5 (55.5%)	3 (33.3%)	1 (11.1%)	0 (0%)	0 (0%)	4.33	1.000
Procedural Inaccuracy	4 (44.4%)	3 (33.3%)	2 (22.2%)	0 (0%)	0 (0%)	4.22	.833
Conceptual Confusion	3 (33.3%)	4 (44.4%)	2 (22.2%)	0 (0%)	0 (0%)	4.11	.782
Factual Inaccuracy	2 (22.2%)	4 (44.4%)	2 (22.2%)	1 (11.1%)	0 (0%)	3.78	.972
Inadequate Attitude & Motivation	2 (22.2%)	2 (22.2%)	4 (44.4%)	1 (11.1%)	0 (0%)	3.56	1.014

Further analysis of the lecturers' open feedback revealed that students struggle with understanding the correct steps to solve limits, particularly in differentiating between limits approaching zero and infinity. They often take "shortcuts" in their learning process. Although there are many reliable calculus textbooks/references available, students tend to favour alternative learning methods like watching YouTube videos, where the content may sometimes be oversimplified or even inaccurate. Many also rely on quick solutions from tools like ChatGPT and free math apps without fully understanding the underlying concepts. Instead of working through the logical steps needed to get a solution, they prefer to use fewer steps, focusing solely on getting the answer rather than grasping the reasoning behind it. Mixing up methods for solving different types of limits and insufficient algebraic skills prevent students from effectively solving the limit problems. Additionally, attitude issues, such as low motivation, a preference for shortcuts, and negligence in engaging with study materials, further exacerbate these difficulties.

4.8 Lecturers' Strategies to Improve Students' Understanding of Limits

Table 12 presents the lecturers' feedback on the strategies to improve students' understanding of limits. The most recommended approach was frequent practice and feedback, which was rated as "very effective" and "more effective" by all respondents (Mean=4.56, S.D.=.527). This is closely followed by emphasising pre-requisite knowledge, which was rated as "very effective" and "more effective" by 88.8% of the respondents (Mean=4.44, S.D.=.726). Addressing misconceptions and common errors and focusing on both conceptual and procedural understanding were also highly rated, with a similar rate of 88.8% considering these strategies as "very effective" and "more effective" (Mean=4.22, S.D.=.667). Additionally, 77.7% of the respondents rated fostering collaborative problem-solving and creating discussion-rich environments as "very effective" and "more effective" (Mean=4.22, S.D.=.833). Although conducting interactive and dynamic lessons was rated the lowest, it received strong support, with 55.5% rating them as "very effective" and "more effective".

Table 12. Strategies to improve students' understanding of limits

Strategies	Very Effective	More Effective	Effective Enough	Less Effective	Not Effective	Mean	S.D.
Frequent Practice & Feedback	5 (55.5%)	4 (44.5%)	0 (0%)	0 (0%)	0 (0%)	4.56	.527
Emphasise Pre-requisite Knowledge	5 (55.5%)	3 (33.4%)	1 (11.1%)	0 (0%)	0 (0%)	4.44	.726
Diagnose Misconceptions & Common Errors	3 (33.4%)	5 (55.5%)	1 (11.1%)	0 (0%)	0 (0%)	4.22	.667
Focus on Conceptual Understanding	3 (33.4%)	5 (55.5%)	1 (11.1%)	0 (0%)	0 (0%)	4.22	.667
Focus on Procedural Understanding	3 (33.4%)	5 (55.5%)	1 (11.1%)	0 (0%)	0 (0%)	4.22	.667
Foster Collaborative Problem-Solving	4 (44.4%)	3 (33.4%)	2 (22.2%)	0 (0%)	0 (0%)	4.22	.833
Create a Discussion-Rich Environment	4 (44.4%)	3 (33.4%)	2 (22.2%)	0 (0%)	0 (0%)	4.22	.833
Interactive & Dynamic Lessons	3 (33.4%)	2 (22.2%)	4 (44.4%)	0 (0%)	0 (0%)	3.89	.928

Further analysis on the lecturers' open feedback disclosed that encouraging discussion sessions and hands-on practices was highlighted as an effective method for engaging students and reinforcing concepts. The use of mind maps to organise and summarise key concepts was seen as beneficial for helping students visualise connections between ideas, enhancing recall, and understanding the broader framework of limits. Summarising the chapter on limits and providing a big-picture visualisation were also highlighted as effective strategies to clarify concepts and relationships. Besides, tools such as Symbolab, Maple, and Quizizz were recommended to illustrate concepts graphically and provide interactive revision opportunities. Incorporating hands-on practice and active discussions, along with showcasing students' work to highlight and correct common mistakes, was also emphasised to encourage students' reflection, develop accuracy, and enhance their understanding.

5.0 DISCUSSION

5.1 Three Types of Common Errors in Solving Limits

The study identified three common types of errors encountered when solving limit problems. Conceptual errors, which stem from misunderstandings of the fundamental concepts and principles related to limits, contributed to the highest error percentage of 67.6%. This was followed by procedural errors (18.9%), which involve incorrect steps or algorithms during the mathematical process, and factual errors (13.5%), which are mistakes in recalling or applying arithmetic facts or properties. The predominance of conceptual errors is consistent with findings by Ghazali and Zakaria (2011), who found that students have a strong grasp of procedures but struggle with the underlying concepts. Lai (2012) highlighted that students facing conceptual errors often exhibit difficulties in translating limit problems into visual or tangible representations, further impeding their comprehension. As emphasised by Brown et al. (2016), conceptual errors frequently arise from a fragmented understanding of calculus fundamentals, making it difficult for students to correctly apply theoretical knowledge.

Although procedural errors are less common than conceptual errors, they still require significant attention as they represent failures in correctly applying known methods to solve problems (Lai, 2012; Brown et al., 2016). Lai (2012) noted that procedural mistakes are particularly common among students who rely on rote memorization rather than conceptual understanding. As emphasised by Baidowi et al. (2023), conceptual errors frequently lead to procedural mistakes. Brown et al. (2016) suggested that procedural errors can be mitigated through structured problem-solving techniques that emphasize logical reasoning and verification steps.

While conceptual and procedural errors receive greater attention in research due to their deep-rooted impact on mathematical understandings, factual errors also play a crucial role in hindering student performance (Lai, 2012; Brown et al., 2016; Giri & Gowramma, 2021). Lai (2012) highlighted that factual errors are particularly prevalent among students who struggle with retaining mathematical properties, leading to repeated mistakes in problem-solving exercises. Similarly, Brown et al. (2016) observed that these errors often arise from inconsistencies in students' prior mathematical knowledge, necessitating frequent review and reinforcement of fundamental concepts. Since factual errors are typically caused by carelessness rather than conceptual misunderstanding (Brown et al., 2016; Lai, 2012), they are relatively easier to correct. However, if ignored, they can disrupt learning, reinforce incorrect habits and reduce students' confidence (Giri & Gowramma, 2021; Lai, 2012).

The study also observed that some students made no attempt to solve problems, suggesting that factors like anxiety, personal issues, lack of preparation, or time pressure may discourage engagement (Hassim & Zainal Abidin, 2020; Lai, 2012).

5.2 Challenges and Impact on Students' Performance

The findings indicated that insufficient pre-requisite knowledge was the most significant challenge that affects students' performance in solving limit problems. This aligns with existing literature, which emphasises the importance of a strong foundation in algebra, trigonometry, and calculus basics for understanding advanced concepts like limits (Baidowi et al., 2023). Students who lack this foundation struggle to connect new concepts with existing knowledge, which increases the likelihood of errors (Jameson et al., 2023). Similarly, Brown et al. (2016) highlighted that deficiencies in foundational knowledge contribute to systematic errors, as students are unable to connect prior mathematical concepts with new calculus principles, ultimately affecting their ability to solve limit problems accurately.

Conceptual confusion, along with procedural and factual inaccuracies, also emerged as major challenges. These challenges often intersect, as inadequate conceptual knowledge often results in students' procedural mistakes, which compounds their difficulties (Villavicencio, 2023). This suggests that effective problem-solving requires not only a solid conceptual foundation but also the ability to execute problem-solving steps accurately (Ghazali & Zakaria, 2011; Nurhayati & Retnowati, 2019). Although issues related to attitude and motivation were deemed less significant, they remain relevant factors in students' performance. Inadequate attitude and motivation towards learning influence students' engagement and persistence in mastering limits (Hassim & Zainal Abidin, 2020).

5.3 Strategies to Improve Students' Understanding of Limits

The findings revealed that students primarily rely on consulting resources, revisiting foundational concepts, and engaging in collaborative learning to improve their understanding of limits, with consulting resources being the most preferred strategy. This preference aligns with Rougeaux and Sharp (2023), who emphasised the importance of utilising various resources, including human and technological tools, to access mathematical concepts and hence offers flexibility and supports deeper conceptual understanding of complex topics.

Revisiting foundational concepts significantly enhances problem-solving skills by reinforcing prior knowledge, which is essential for developing a more comprehensive and accurate understanding of limits (Wakhata et al., 2023, Jameson et al., 2023). The importance of collaborative approaches, such as group discussions and peer study, also aligns with prior studies, which highlight the value of shared problem-solving and peer interaction in fostering a deeper understanding. These approaches allow students to engage with diverse perspectives and collaboratively tackle challenging problems (Villavicencio, 2023; Ghazali & Zakaria, 2011).

Although practising additional exercises was moderately utilised, it is often discussed in prior studies for its potential to enhance memory retention and improve problem-solving skills (Wakhata et al., 2023; Denbel, 2014). However, the limited student engagement in asking questions, whether in class or during office hours, reflects barriers such as fear of judgment, lack of confidence, or emotional issues (Schafer & O'Neill, 2023). This reluctance can hinder the clarification of misconceptions and obtaining direct feedback, reinforcing the need for instructors to cultivate a supportive and inclusive environment where students feel encouraged to actively participate (Hassim & Zainal Abidin, 2020).

Lecturers, on the other hand, strongly emphasised the importance of frequent practice and feedback as the most effective strategies, underscoring their role in enhancing understanding and identifying errors early (Nurhayati & Retnowati, 2019). Equally important was the need to address pre-requisite knowledge gaps (Baidowi et al., 2023). Addressing misconceptions and common errors through conceptual and procedural clarity was also prioritised to correct flawed reasoning and enhance problem-solving skills (Wakhata et al., 2023). While collaborative problem-solving and interactive lessons were slightly lower in priority, they remain effective in promoting students' critical thinking and engagement (Villavicencio, 2023).

6.0 CONCLUSION

This study highlights three main types of errors students commit when solving limit problems in calculus: conceptual errors (67.6%), procedural errors (18.9%), and factual errors (13.5%). While most students are positive about learning limits of functions, a significant number reported being hesitant or moderately confident when tackling limit problems. Insufficient pre-requisite knowledge, conceptual confusion, as well as procedural and factual inaccuracies were identified as the most significant challenges in understanding limits. Students commonly relied on consulting resources, revisiting foundational concepts, and collaborative learning to improve their understanding. On the other hand, lecturers emphasised the importance of frequent practice, constructive feedback, and addressing misconceptions and common errors to enhance students' comprehension and performance in solving limit problems.

For academic practitioners, the findings highlight the necessity of integrating targeted interventions, such as guided problem-solving exercises, error-based feedback, and technology-enhanced learning tools like interactive simulations and AI-driven tutoring systems, to effectively address learning challenges. For policymakers, the findings highlight the need for curriculum and assessment reforms, along with professional development programmes that equip educators with more effective teaching methodologies.

Although this study provides valuable insights, its focus is limited to a specific group of students and lecturers, which may restrict the generalisability of the findings. Future research should include a larger and more diverse sample across various institutions and disciplines to validate and expand on these findings. Additionally, investigating the long-term effectiveness of recommended strategies, such as frequent practice and collaborative learning, could provide deeper insights. Exploring the role of technology and innovative teaching tools in addressing conceptual and procedural challenges could further enhance students' learning outcomes. These efforts could help to develop more effective approaches for improving understanding and performance in solving limit problems in calculus.

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